1. Electricity travels at the speed of light. This means the electricity in the wires quickly dissipates if the power source is removed. Let’s see how much energy might be “stored” in the wire using a simple pipe model where the energy stored in the pipe is the power times the propagation time from one end of the wire to the other. Consider a 1 GW power plant that serves customers that are 100 miles from the plant. You may assume that the electricity travels at the speed of light in a vacuum.

   (a) Compute the energy stored in the wire in units of kWh. Compare this amount of stored energy to an energy use around your house.

   (b) In reality the power is sent as an alternating current. Compute the energy stored in the wire as the voltage passes through zero volts.

Sophisticated power system switching equipment attempts to open and close circuits as the voltage passes through zero. Otherwise, lightning-like high-voltage arcs will jump across the small gap just as the switch opens and closes leaving weld marks that accumulate over time and deforms the equipment until it is unusable or worse has a catastrophic failure.

2. The first major electric power system started producing 75 MW of power in 1896 at Niagara Falls. What made it useful was that some of the power could be transported from the falls to the city of Buffalo 32km away. In order to have low enough transmission losses, low resistance wire was needed. In general thicker wire has lower resistance. The residence of a wire with circular cross section is given by

\[ R_{DC} = \frac{4L\rho}{\pi D^2} \]

where \( L \) is the length of the wire, \( D \) is the diameter of the wire and \( \rho \) is the resistivity of the wire. Note that the unit of resistance is the ohm (\( \Omega \)) and it has the following equivalence: \( \Omega = \text{Volt}^2/\text{Watt} \).

However for AC power, the electricity does not travel throughout the body of the wire and instead travels mostly near the surface due to the skin effect. The skin depth, \( \delta \), is a measure of how near the surface of the wire that the electricity flows. It is a function of the material and the frequency. For thin wires this is not significant. But for thicker wires it can make a large difference. Taking into account for the skin depth, the resistance of a wire is:

\[ R_{AC} = \begin{cases} R_{DC} & \text{if } D < 3\delta \\ R_{DC} \left( \frac{D}{3\delta} + 0.26 \right) & \text{otherwise} \end{cases} \]

Properties for copper and aluminum (common wire materials) are:
3. A wind turbine is often rated with statements such as “provides enough power for 1000 homes”. This implies that, since there are about 125M homes in the U.S., that 125,000 turbines will power the country. Look at the data in EIA Electric Power Annual for residential energy use versus other customers. If a single turbine powers 1000 homes, estimate how many turbines would be required to power all sectors of the country.

4. A three phase circuit consists of three wires emanating from a generator carrying currents:

\[
I_1(t) = A_1 \cos(2\pi ft), \\
I_2(t) = A_2 \cos(2\pi ft + \frac{2\pi}{3}), \text{ and} \\
I_3(t) = A_3 \cos(2\pi ft + \frac{4\pi}{3}).
\]

These currents pass through three separate loads and are tied to a single, so-called, neutral wire that carries the sum of the currents back to the generator. (hint: \(\cos(A + B) = \cos A \cos B - \sin A \sin B\) and \(\cos^2 A = \frac{1}{2}(1 + \cos 2A)\))

(a) Show that if \(A_1 = A_2 = A_3\) then the sum of the currents is zero. In other words, the neutral wire can be much smaller than the three-phase power delivery wires.

(b) Show that if \(A_1 = A_2 = A_3\) and the wires are powering equal loads of resistance \(R\), then the total power is a constant where

\[
\text{Total Power} = (I_1^2(t) + I_2^2(t) + I_3^2(t))R.
\]

(c) Now if \(A_1 = A_2\) and \(A_3 = 0\), discuss how this unbalanced load compares to the balanced load’s constant power and zero neutral wire current.