The Capacity of Wireless Networks

Piyush Gupta, Student Member, IEEE, and P. R. Kumar, Fellow, IEEE

Presented by Ketan Kambli
Motivation & Background:

- Ad-hoc wireless network
  - Cooperate in routing each other’s data packets
- Are there limits?
- Equivalent of nearest-neighbor communication
... Motivation & Background:

• “A wireless medium is more like a cocktail party than a telephone call.”  P.R.Kumar

• Generalization of Channel coding theorem

• Looking at really huge systems

• Analogy in thermodynamics

• what you can say about wireless networks in the aggregate
Setup

- $n$ nodes are located in a region of area 1 m$^2$
- Each node can transmit at $W$ bits/sec over a common wireless channel
- Packets are sent from node to node in a multi-hop fashion
Modeling the networks

• Arbitrary Networks
  – node locations, destinations of sources, and traffic demands, are all arbitrary
  – Arbitrary range or power level for each transmission

• Random Networks
  – n nodes are randomly located either on the surface of a 3-dimensional sphere of area 1 sq. meter, or in a disk of area 1 sq. meter in the plane
  – randomly chosen destination
when a transmission is received successfully?

• The Protocol Model
  – Considers distance
  – Non interference protocol is assumed

• The Physical Model
  – More related to physical layer
  – Considers minimum SIR required
Protocol Model

• \(X_i\) transmits to \(X_j\) successfully on subchannel \(m\) if

\[
\left| X_k - X_j \right| \geq (1+\Delta) \left| X_i - X_j \right|
\]

• for every other node \(X_k\) simultaneously transmitting over the same subchannel

• Delta is the guard zone
• For the Random Network model, as all nodes transmit at a common power, thus having a common range $r$:

$$|X_i - X_j| \leq r$$
Physical Model

- \( \{X_k; k \in \Gamma\} \) being the subset of simultaneously transmitting nodes over a same subchannel, with power level \( P_k \); then transmission from \( X_i \) to \( X_j \) is successful if:

\[
\frac{P_i}{|X_i - X_j|^\alpha} \geq \beta \quad ; \quad N + \sum_{k \in \Gamma, k \neq i} \frac{P_k}{|X_k - X_j|^\alpha} \geq \beta
\]

**Arbitrary Network**

\[
\frac{P}{|X_i - X_j|^\alpha} \geq \beta \quad ; \quad N + \sum_{k \in \Gamma, k \neq i} \frac{P}{|X_k - X_j|^\alpha} \geq \beta
\]

**Random Network**
The Transport Capacity of Arbitrary Networks

• let us say that the network transports one bit-meter when one bit has been transported a distance of one meter toward its destination

• This sum of products of bits and the distances over which they are carried is a valuable indicator of a network’s transport capacity
• Big O notation

• Theta notation
Results for Arbitrary Networks

• Under protocol model

$$\Theta(W \sqrt{n})$$

• Under physical model $cW \sqrt{n}$ is feasible but $c'Wn^{a-1/a}$ is not for appropriate $c$, $c'$. 
The Transport Capacity of Random Networks

• In case of protocol model for both surface of sphere and planar disk the order of the throughput capacity is

\[ \lambda(n) = \Theta \left( \frac{W}{\sqrt{n \log n}} \right) \]

• For the physical model, a throughput of

\[ \lambda(n) = \frac{cW}{\sqrt{n \log n}} \]

bit per second is feasible, while

\[ \lambda(n) = \frac{c'W}{\sqrt{n}} \]

is not for appropriate c and c'.
Implications:

• Understanding the results

• Scaling

• What are the reasons for these limits?
  – Hot spots?

• Will splitting the channel in sub channels help?
• Fundamentally, it is the need for every node all over the domain to share whatever portion of the channel it is utilizing with nodes in its local neighborhood that is the reason for the constriction in capacity.

• Splitting the channel into several subchannels does not change any of the results.

• The order of throughput capacity is the same as on the surface of the sphere, it shows that the cause of the throughput constriction is not the formation of hot spots, but is the pervasive need for all nodes to share the channel locally with other nodes.
Some issues

• Protocol model:
  – If perfect node location information is not available, or if nodes move, or traffic demands are not known, then the capacity can only be even smaller

• Application of results to infrastructure mode

• Number of Relay nodes
Implications for designers

• Since the throughput furnished to each user diminishes to zero as the number of users is increased, perhaps networks connecting smaller numbers of users, or featuring connections mostly with nearby neighbors, may be more likely to be find acceptance.
Questions?