Public Safety and Commercial Spectrum Sharing via Network Pricing and Admission Control

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Abstract— **Public safety and commercial communications have distinct performance objectives. The former requires a high availability (i.e., low blocking), both in normal times and during a disaster, whereas the latter requires a high network revenue performance in normal times. Therefore, public safety and commercial communications traditionally use separate networks, which results in either spectrum inefficiency or unsatisfactory performance. In this paper, we propose a public safety and commercial spectrum sharing strategy via intelligent network pricing and call admission control. We provide a precise characterization of the performance objectives of such a sharing strategy, which is to maximize the commercial revenue while guaranteeing a low blocking probability to public safety calls. We demonstrate through analysis and simulation that our sharing strategy simultaneously satisfies the objectives of both types of communications.**

Index Terms— **Call admission control, network pricing, public safety and commercial communications, spectrum sharing**

I. Introduction

IN THIS paper, we address the problem of spectrum sharing between public safety and commercial communications. Public safety personnel rely heavily on radio communications, which are built on radio spectrum resources that consist of channels within public safety radio bands. However, these resources are also highly valuable for providing commercial communications. The proper use of spectrum for both public safety and commercial communications is therefore critical.

Public safety and commercial communications differ in several respects. Public safety calls (a single push-to-talk on period is considered a call [1]) tend to be shorter and more frequent, and the demand peaks at different times. Additionally, public safety users communicate free of charge on an as needed basis. Its most important performance measure is the high availability (i.e., the low blocking probability) since missed connections can mean loss of life and property. By contrast, commercial users must pay for their usage and the performance objective of commercial service providers is to maximize network revenue. Furthermore, the commercial users' traffic demand is affected by the connection price charged by commercial operators. The traditional approach to meeting these different objectives is to allocate separate bands for public safety and commercial use. However, emergencies can create surges in demand that are many times that of normal usage and are concentrated in both time and space. Public safety needs spectrum precisely for such emergencies. Meanwhile, commercial spectrum users can adjust demand through pricing to match resources, designing for normal day busy hours and ignoring extreme usage cases such as a public sports or performance events. Allocating enough dedicated bandwidth to cover public safety needs through all emergencies means that the public safety band is typically underutilized, while commercial users are crowded out, leading to higher user fees and lower revenues for commercial operators. As a result, the public safety allocations, though high relative to comparable commercial spectrum usage [2], are typically insufficient and public safety users suffer radio congestion during emergencies. Thus, an intelligent spectrum sharing mechanism is needed to best satisfy the requirements of both public safety and commercial communications.

Although the rationale for sharing spectrum resources between public safety and commercial communications is clear, the design of an efficient sharing mechanism is not trivial. In this paper, we propose a public safety and commercial spectrum sharing strategy via network pricing and call admission control (CAC). We consider trunked radio resources, where multiple channels are shared by public safety users and commercial users. To be concrete, we assume a direct sharing of commercial cellular system resources that would co-mingle public safety and commercial calls, as proposed in [3], [4]. Specifically, we consider a two-stage control process. In the first stage, a sound static commercial network pricing policy is used to set the overall commercial traffic load. In the second stage, an intelligent state-dependent dynamic CAC policy is used to control the call admission or rejection decision. Because the price policy is static, the price information needs to be communicated to the user once. Meanwhile, although a dynamic CAC policy is state-dependent and adapts its policy according to changing network conditions, the CAC function is internal to a network and is transparent to users. Therefore, the proposed sharing strategy can respond to changing network conditions and achieve a sound performance, yet avoid frequent exchange of information between the network controller and the users.

To address the spectrum sharing problem, we define the performance objective to be the maximal revenue performance given the constraint of satisfying the blocking probability requirements for the public safety calls. We quantify the effect of our sharing mechanism on commercial user prices and commercial provider revenues relative to the segregated public safety and commercial networks that use the same amount

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of total spectrum resources. We also address the implications on public safety users, commercial operators, and commercial users. We demonstrate that, with a properly designed pricing and CAC policy pair, our two-stage spectrum sharing strategy performs well in that it provides better commercial revenue and meets public safety blocking requirements in more cases than (a) segregated bands, or (b) using trivial complete band sharing. Furthermore, consumers benefit since the price to consumers is lower than that in the segregated or that in the trivially shared cases, and this performance is achieved without using preemption of ongoing commercial calls.

The remainder of this paper is organized as follows. Section II reviews related work, and Section III formulates the problem. In Section IV, we present our spectrum sharing mechanism and analyze the performance of the shared networks via various pricing and CAC combinations. We describe the computation methods for pricing and CAC policies in Section V. We then give numerical examples in Section VI. We discuss the practical implications of our results and address direction for future work in Section VII. Finally, Section VIII concludes the paper.

II. Related Work

Simple spectrum sharing mechanisms are currently used or have been proposed [5]. However, these either do not satisfy the high-availability requirements for public safety or use spectrum resources inefficiently. For instance, one approach would be to use a complete sharing of the spectrum. Such sharing will yield lower overall blocking but provides no protection to public safety users in a disaster (where there is a surge in both public safety and commercial user traffic). A solution proposed by the FCC Spectrum Policy Task Force is to have bandwidth that is shared between public safety and commercial users [6]. In normal times, the commercial users can use more of the spectrum, while during emergencies the public safety users can use more of the spectrum. Intuitively, dynamic sharing provides a trunking effect that should produce a net benefit to both users, but it is not clear what mechanism provides the most benefit.

Using pricing to adjust the network traffic demand (i.e., price-based traffic volume control) and using CAC for network control were studied earlier. The key objective of price-based traffic volume control is to use the price to determine the optimal traffic volume, as characterized by the connection arrival rate [7]–[11]. Thus, the price is a critical factor in determining the number of users requesting services at a certain time. With a price-based traffic volume assignment, both *static pricing* and *dynamic* (or *congestion-dependent*) *pricing* were studied. A static pricing is independent of the network state (or, equivalently, the congestion level), but a dynamic one varies accordingly. Meanwhile, the key objective of network CAC research is to achieve a performance gain through an intelligent CAC policy, see [12]–[15] and references therein. A trivial baseline CAC policy is the so-called *greedy* (or *complete sharing*) policy, whereby a network always accepts a connection request as long as bandwidth is available. By contrast, a *dynamic* CAC policy bases its acceptance or rejection of a connection request on both the available bandwidth and the system state at the decision time. Therefore, a dynamic CAC policy can reject a connection request even when there is available bandwidth if a connection is deemed to be less valuable than the potential future connections.

In our earlier work [11], we studied the relative roles of network pricing and CAC in network revenue optimization for a network without any traffic loss rate requirement. In this paper, we apply network pricing and CAC control for public safety and commercial spectrum sharing. Moreover, in contrast to the loss rate unconstrained network revenue optimization problem in [11], the spectrum sharing problem requires maximizing network revenue while meeting the traffic loss rate requirements of public safety calls. Therefore, a computational approach different from the one in [11] is used to solve the optimal constrained dynamic CAC policy.

III. Problem Formulation

In this section, we formulate a simplified model that captures the main elements of the problem. We first introduce the network bandwidth sharing model and its support of multi-class traffic. Secondly, we describe the representative network operating conditions for such a shared network, where a network condition is characterized by the traffic load. We then describe the statistical characteristics of the network traffic and define the system state. Subsequently, we introduce the framework of our spectrum sharing strategy. Finally, we formulate the performance objective of the network with the shared spectrum for public safety and commercial communications.

A. Network model

We denote the bandwidth capacity of a general network by C. We treat the spectrum resource as a single shared resource which supports both public safety and commercial communications. For a fair comparison, we first denote by C_{ps} and C_{com} the bandwidth capacities of a segregated public safety network and of a commercial communication network, respectively. We then let the bandwidth of the shared spectrum be equal to the sum of the segregated public safety and commercial networks:

$$C_{\mathrm{t}} = C_{\mathrm{ps}} + C_{\mathrm{com}}.$$

Public safety and commercial users attempt to access this shared bandwidth to place *calls*. These calls can belong to multiple traffic classes in the form of narrowband voice calls or broadband calls for data/video types of applications. We therefore assume a set $\mathcal{I}_{\mathrm{t}} = \{1, ..., I\}$ of call classes in the shared network partitioned into public safety and commercial call types. In particular, we denote by $\mathcal{I}_{\mathrm{ps}}$ and $\mathcal{I}_{\mathrm{com}}$ the class index sets for the public safety and commercial calls, respectively, where:

$$\mathcal{I}_{\mathrm{ps}} \subset \mathcal{I}_{\mathrm{t}}, \qquad \mathcal{I}_{\mathrm{com}} \subset \mathcal{I}_{\mathrm{t}}, \qquad \mathcal{I}_{\mathrm{ps}} \cap \mathcal{I}_{\mathrm{com}} = \emptyset,$$

and,

$$\mathcal{I}_{\mathrm{t}} = \mathcal{I}_{\mathrm{ps}} \cup \mathcal{I}_{\mathrm{com}}.$$

To simplify the analysis, we consider a centralized controller for setting the network control policy, although equivalent distributed controllers are possible. We also assume that the spectrum allocation is for a localized interference area similar to a single cell in a cellular radio system.

B. Network conditions

The network traffic load is drastically different during normal operation and special network conditions. A key requirement for any public safety and commercial spectrum sharing mechanism is that it satisfies the network performance objective under varying network operating conditions. In particular, changes in network conditions, such as induced by a disastrous event, are often unpredictable. Therefore, network control needs to be able to adapt to network conditions.

To characterize different operating conditions, we denote by $\mathcal{M}$ the set of these conditions and characterize a specific condition by the corresponding traffic *load* of each class. We assume that the traffic service rate is independent of a specific network condition and that the network condition is characterized by its *arrival rate*. The set of network conditions could be a continuous space covering the range of possibilities. For simplicity, we assume that it consists of a finite enumeration of representative conditions $m \in \mathcal{M}$. Some representative network conditions can be categorized as following:

- *Normal (*norm*)*: both public safety and commercial traffic are at a defined normal level;
- *Emergency (*emer*)*: public safety traffic is at a high level while commercial traffic is at a normal level;
- *Disaster (*dstr*)*: both public safety and commercial traffic are at a high level;
- *Public Event (*evnt*)*: public safety traffic is at a normal level and commercial traffic is at a high level.

We emphasize that the network adapts to the network conditions as they arise and does not have a pre-computed policy for each condition. The above representative conditions will be used to demonstrate what network performance we might expect for typical cases of concern to public safety users.

C. Network traffic

We make the following assumptions on the network traffic. The traffic of each class arrives according to a Poisson process and the service times are exponentially distributed. Each traffic class $i \in \mathcal{I}_{\mathrm{t}}$ has a transmission rate (i.e., bandwidth) x_i, and a mean service rate μ_i. The arrival rate for traffic class i depends on the network operating condition and is denoted by $\lambda_{i,m}$, where $m \in \mathcal{M}$ is the network condition. Public safety call class $i \in \mathcal{I}_{\mathrm{ps}}$ has a predefined fixed arrival rate $\lambda_{i,m}$ for each condition $m \in \mathcal{M}$, and there is no charge to public safety calls. By contrast, the network provider charges a static price $u_i, i \in \mathcal{I}_{\mathrm{com}}$, for every accepted class-i commercial call. Moreover, such a static pricing u_i uniquely determines the arrival rate of commercial calls $\lambda_{i,m}$. There are no demand substitution effects among different traffic classes. Letting $\lambda_{i,m}(\cdot)$ represent a given continuous and strictly decreasing arrival function, or in economic terms a *demand* function, we have:

$$\begin{aligned} &\lambda_{i,m} && \text{if } \ i \in \mathcal{I}_{\mathrm{ps}}, \\ &\lambda_{i,m} = \lambda_{i,m}(u_i) && \text{if } \ i \in \mathcal{I}_{\mathrm{com}}, \end{aligned}$$

where $0 \le u_i \le u_{i,\max}$ and $\lambda_{i,m}(u_{i,\max}) = 0$ for $i \in \mathcal{I}_{\mathrm{com}}$. Such demand functions for the commercial calls agree with classic economic theory on the relationship between the amount of consumer demand of a good and the corresponding price [16].

D. System state definition

For a network with capacity $C = C_t$, we define the system state generically for all network conditions, where the system state is the combination of the system configuration and the transition-triggering traffic events. For a particular network condition $m \in \mathcal{M}$, the corresponding traffic parameters shall apply. Hence, we omit the subscript m to simplify notation. We let n_i denote the number of ongoing class-i calls in the system, and the system configuration is defined to be the number of calls of each class in the system, i.e., $n = (n_1, ..., n_I)$. We also assume that the number of state transitions within a finite time interval is finite and there is only one traffic event taking place at any instant. Let $\boldsymbol{e}_i$ denote a $(1 \times I)$ unit vector whose i^{th} element equals 1 and the other $I-1$ elements equal 0. Subsequently, we can use $\omega = \boldsymbol{e}_i$ and $\omega = -\boldsymbol{e}_i$ to denote the arrival and the departure of a class-i call, respectively. Therefore, the configuration space $\mathcal{N}$ and the event space Ω can be defined as follows:

$$\begin{aligned} \mathcal{N} &= \Big\{ n \mid \sum_{i=1}^{I} n_i x_i \le C_{\mathrm{t}} \Big\}, \\ \Omega &= \Big\{ \ \omega \mid \omega \in \{\pm \boldsymbol{e}_i\} \ \Big\}. \end{aligned}$$

Due to the finite capacity constraint and the finite number of traffic classes, the system is assumed to constitute a state space $\mathcal{S}$ with a finite number of elements, as defined in (1):

$$\begin{aligned} \mathcal{S} = \Big\{ & s = (n, \omega) \mid n \in \mathcal{N}, \ \omega \in \Omega \\ & \text{and } \omega \ne -\boldsymbol{e}_i \text{ if } n_i = 0 \Big\}. \end{aligned} \tag{1}$$

We also define $\mathcal{S}_j^A$ in (2) as the set of states whose traffic event is the arrival of a class-j call:

$$\mathcal{S}_j^A = \Big\{ s = (n, \omega) \mid n \in \mathcal{N}, \ \omega = \boldsymbol{e}_j \Big\}. \tag{2}$$

E. Framework of the sharing strategy

When the network employs a trivial greedy CAC policy to satisfy the low blocking (loss) rate requirements of public safety calls in all network conditions, it requires either a large amount of the shared spectrum, or high commercial prices to drive down the commercial service demand. However, allocating large spectrum is costly given that spectrum resource is expensive, and high call prices lead to low commercial service demand and hence to loss of the network provider's revenue.

To simultaneously maximize the revenue performance while guaranteeing the required loss rate of public safety calls during all network conditions with the shared spectrum C_{t}, we propose a network pricing- and CAC-based two-stage spectrum sharing strategy: 1) use a sound static pricing policy to set the *right* amount of commercial traffic; 2) apply an optimal dynamic (i.e., state-dependent) CAC policy for admission control. This strategy is to contrast with selecting the commercial prices arbitrarily and using the greedy CAC policy.

It is motivated by our observation in [11] that the impact of network pricing and CAC can be viewed as a unified two-stage network control process, where pricing affects the amount of traffic presented to a network and CAC subsequently affects the amount of traffic accepted into the network.

Moreover, the static pricing vector of the commercial calls, denoted by u, is independent of the network condition: commercial network users are charged a fixed price for a call belonging to a particular class, regardless if the network is in a normal operational mode or if it is handling a surge of service demand under special conditions (e.g., emergency, disaster, etc.). Meanwhile, unlike pricing, CAC can be adaptive and follow network conditions. We denote the CAC policy for network condition $m \in \mathcal{M}$ by π_m. The network traffic parameters (e.g., arrival and service rates, etc.) are monitored and estimated periodically and then are used to compute the optimal dynamic CAC policy for the current condition.

We note that if our designed prices are combined with the greedy CAC, the public safety call blocking (loss) rate requirements may not be met. Instead, such requirements are ensured only when the pricing policy is combined with a constrained dynamic CAC policy that incorporates the call level blocking requirements. Additionally, with this framework, the pricing information needs to be communicated to the users only once, while the operation of a dynamic CAC policy is internal to a network and can be adapted to network conditions transparent to users. Therefore, the network services offering based upon our framework is simple without the need to frequently exchange information between the network operator and users.

We also make the following assumption. It is general but important to ensure the validity of our sharing strategy.

Assumption 1: The total capacity $C_{\rm t}$ is large enough so that in any network condition $m \in \mathcal{M}$, the traffic blocking (loss) rate constraint can be met if there is no commercial load on the shared network and all the spectrum $C_{\rm t}$ is shared.[1]

With this assumption, we can establish the existence of pricing and CAC policies that satisfy the blocking (loss) rate requirements of public safety calls.

Proposition 1: When Assumption 1 holds, for any commercial price vector u, in any network condition $m \in \mathcal{M}$, there exists a CAC policy π_m such that the loss rate requirements of public safety calls are satisfied.

Proof

Assumption 1 states that, for every $m \in \mathcal{M}$, if $\lambda_{i,m} = 0$ for all $i \in \mathcal{I}_{\rm com}$, then $L_{i,m} \le L_{i,{\rm req}}$ for all $i \in \mathcal{I}_{\rm ps}$. Therefore, for any commercial price vector u and the corresponding traffic arrival rate vector $\lambda_m(u)$, we have at least one CAC policy, which is to reject all commercial traffic, so that the blocking (loss) rate requirements of public safety calls are satisfied. Hence the result. *Q.E.D.*

F. Performance objective

The task of the network controller is to select a sound price for each traffic class and to identify an intelligent CAC policy to obtain the optimal long term revenue while satisfying the loss rate requirements of public safety calls. We use $\mathcal{U} = \{u \mid u_i \in [0, u_{i,max}],\ i \in \mathcal{I}\}$ to denote the set of all pricing policies, and $\Pi_m = \{\pi_m\}$ the set of all CAC policies under network condition m. We also denote the set of average revenue performance under network condition m by $\mathcal{R}_m = \{R_m(u, \pi_m) \mid u \in \mathcal{U}, \pi_m \in \Pi_m\}$. Let $L_i\big[t, \lambda_{{\rm ps},m}, \lambda_{{\rm com},m}(u), \pi_m\big]$ denote the loss probability of class-i calls at time t for a given pricing policy u and a CAC policy π_m. We note that $\mathcal{R}_m$ and L_i depend on the network bandwidth capacity C. For a shared network where $C = C_t$ here, C is dropped from $\mathcal{R}_m$ and L_i to simplify notation. For general pricing and CAC policies, the network revenue under network condition $m \in \mathcal{M}$ can be expressed as in (3):

$$R_m(u,\pi_m) = \lim_{T\to\infty} \frac{1}{T} \sum_{i\in\mathcal{I}_{\rm com}} E\Big\{ \int_0^T \lambda_{i,m}\Big(u_i(t)\Big) u_i(t)\cdot \Big[\, 1 - L_i\Big(t, \lambda_{{\rm ps},m}, \lambda_{{\rm com},m}(u), \pi_m\Big) \Big] dt \Big\}. \quad (3)$$

Under a specific network condition $m \in \mathcal{M}$, when a static pricing policy is applied, the mean connection arrival rate of class-i is independent of time. Therefore, with a static pricing policy and a time-invariant, state-dependent CAC policy, the traffic loss process is stationary. Hence, we can simplify notation by letting $L_{i,m}(u,\pi_m) = L_i\big[t, \lambda_{{\rm ps},m}, \lambda_{{\rm com},m}(u), \pi_m\big]$, so that the long-term average revenue under network condition $m \in \mathcal{M}$ simplifies to:

$$R_m(u,\pi_m) = \sum_{i\in\mathcal{I}_{\rm com}} \lambda_{i,m}(u_i) u_i \big[\, 1 - L_{i,m}(u,\pi_m)\, \big]. \quad (4)$$

As stated earlier, the performance objective of a spectrum sharing mechanism is to best reconcile the requirements of both public safety and commercial profitability. More specifically, it requires a guaranteed maximum public safety call blocking probability and a high network revenue performance. Let $P(m)$ be the probability that the network is in network condition m and $L_{i,{\rm req}}$ is the maximal allowed blocking rate for class-i public safety calls. The network performance objective can then be expressed as:

$$R^* = \sum_{m\in\mathcal{M}} P(m) \max_{u,\pi_m} R_m(u,\pi_m) \quad (5)$$

$$\text{subject to} \quad L_{i,m}(u,\pi_m) \le L_{i,{\rm req}}, \quad \forall\ i \in \mathcal{I}_{\rm ps}, \quad m \in \mathcal{M}.$$

The maximum is taken over possible pricing vectors u and CAC policies π_m. Let R^*_m, $m \in \mathcal{M}$, denote the maximal value of $R_m(u,\pi_m)$ over all possible pairs (u,π_m), then,

$$\begin{aligned} R^*_m &= \max_{u,\pi_m} R_m(u,\pi_m) \\ &= \max_{u,\pi_m} \sum_{i\in\mathcal{I}_{\rm com}} \lambda_{i,m}(u_i) u_i \big[1 - L_{i,m}(u,\pi_m)\big]. \end{aligned} \quad (6)$$

We assume that the commercial operator, when setting the price vector u, is primarily interested in optimizing over usual and known conditions. Certain conditions, such as a disaster, are rare and their extent is highly variable. Therefore, the commercial operator does not consider these conditions in optimizing revenue. The optimal price vector is determined

[1] That is, for every $m \in \mathcal{M}$, if $\lambda_{i,m} = 0$, for all $i \in \mathcal{I}_{\rm com}$, then there exists a policy π_m, such that $L_{i,m}(u = u_{i,max}, \pi_m) \le L_{i,{\rm req}}$ for all $i \in \mathcal{I}_{\rm ps}$, in the shared network with capacity $C = C_{\rm t}$.

from a single Normal condition indicated as $m = \text{norm}$. Thus, the objective function (5) can be simplified to:

$$\begin{aligned} R^* &= \max_{u,\pi_{\text{norm}}} R_{\text{norm}}(u, \pi_{\text{norm}}) \\ &= \max_{u,\pi_{\text{norm}}} \sum_{i\in\mathcal{I}_{\text{com}}} \lambda_{i,\text{norm}}(u_i) u_i \big[1 - L_{i,\text{norm}}(u, \pi_{\text{norm}})\big] \\ &\text{subject to } \; L_{i,m}(u,\pi_m) \le L_{i,\text{req}}, \forall \;\; i \in \mathcal{I}_{\text{ps}}, \;\; m \in \mathcal{M}. \end{aligned} \tag{7}$$

IV. Performance Analysis of the Spectrum Sharing Strategy

To illustrate our spectrum sharing strategy, we first define a set of pricing and CAC policies in this section, and then analyze the performance of various combinations of such policies. We demonstrate that a sound pricing and CAC pair outperforms the alternative trivial policy choices.

A. Definition of pricing and CAC policies

We first define our notation for the CAC policy as follows:

- π_u^*: for a given pricing policy u, the optimal CAC policy that maximizes revenue performance while guaranteeing $L_{i,m}(u, \pi_u^*) \le L_{i,\text{req}}$ for $i \in \mathcal{I}_{\text{ps}}$ and $m \in \mathcal{M}$.[2]
- π^g: the trivial greedy CAC policy for any given pricing.

Now, we define a set of pricing policies. Let $(u_a^*, \pi_{u_a^*}^*)$ be the joint optimal pricing and CAC policies that maximize the long term revenue performance while satisfying the loss rate constraints in a shared network, i.e.,

$$\begin{aligned} \big(u_a^*, \pi_{u_a^*}^*\big) &= \arg\max_{u,\pi} R_{\text{norm}}(u, \pi) \\ &= \arg\max_{u,\pi} \sum_{i\in\mathcal{I}_{\text{com}}} \lambda_{i,\text{norm}}(u_i) u_i \big[1 - L_{i,\text{norm}}(u, \pi)\big] \\ &\text{subject to } \; L_{i,\text{norm}}(u,\pi) \le L_{i,\text{req}}, \;\; \forall \; i \in \mathcal{I}_{\text{ps}} \\ &\text{where } \quad C = C_{\text{t}}. \end{aligned} \tag{8}$$

Let u_b^* be the optimal pricing given the greedy CAC policy π^g while the loss rate requirements $L_{i,\text{req}}$ are not guaranteed:

$$\begin{aligned} u_b^* &= \arg\max_{u} R_{\text{norm}}(u, \pi^g) \\ &= \arg\max_{u} \sum_{i\in\mathcal{I}_{\text{com}}} \lambda_{i,\text{norm}}(u_i) u_i \big[1 - L_{i,\text{norm}}(u, \pi^g)\big] \\ &\text{where } \quad C = C_{\text{t}}. \end{aligned} \tag{9}$$

Let u_c^* be the optimal pricing given the greedy CAC policy π^g while guaranteeing the meeting of the loss rate requirements:

$$\begin{aligned} u_c^* &= \arg\max_{u} R_{\text{norm}}(u, \pi^g) \\ &= \arg\max_{u} \sum_{i\in\mathcal{I}_{\text{com}}} \lambda_{i,\text{norm}}(u_i) u_i \big[1 - L_{i,\text{norm}}(u, \pi^g)\big] \\ &\text{subject to } L_{i,\text{norm}}(u,\pi^g) \le L_{i,\text{req}}, \forall \;\; i \in \mathcal{I}_{\text{ps}}, \\ &\text{where } \quad C = C_{\text{t}}. \end{aligned} \tag{10}$$

[2]Strictly speaking, π_u^* and π^g should be defined as $\pi_{u,m}^*$ and π_m^g, respectively, to indicate that the CAC policy depends on the network condition m. However, we drop the subscript m to simplify notation when the network condition associated with a particular CAC policy is obvious.

TABLE I
PRICING AND CAC DEFINITIONS

Notat.	Definition
π_u^*	for a given pricing policy u, the optimal CAC policy that maximizes the revenue performance while guaranteeing $L_{i,m} \le L_{i,\text{req}}$ for $i \in \mathcal{I}_{\text{ps}}, m \in \mathcal{M}$
π^g	a trivial greedy CAC policy for any given pricing
u_a^*	optimal u for the shared network given $\pi_{u_a^*}^*$
u_b^*	optimal u for the shared network given π^g with no guarantee for $L_{i,m}, i \in \mathcal{I}_{\text{ps}}$
u_c^*	optimal u for the shared network given π^g while guaranteeing $L_{i,m} \le L_{i,\text{req}}, i \in \mathcal{I}_{\text{ps}}$
u_d^*	optimal u for the segregated commercial network given π^g
u_∞^*	optimal u for the shared network assuming $C_{\text{t}} = \infty$

Let u_d^* be the optimal pricing for the segregated commercial network given the greedy CAC policy π^g. That is,

$$\begin{aligned} u_d^* &= \arg\max_{u} R_{\text{norm}}(u, \pi^g) \\ &= \arg\max_{u} \sum_{i\in\mathcal{I}_{\text{com}}} \lambda_{i,\text{norm}}(u_i) u_i \big[1 - L_{i,\text{norm}}(u, \pi^g)\big] \\ &\text{where } \quad C = C_{\text{com}}. \end{aligned} \tag{11}$$

Finally, let u_∞^* be the optimal pricing given infinite bandwidth:

$$\begin{aligned} u_\infty^* &= \arg\max_{u} \sum_{i\in\mathcal{I}_{\text{com}}} \lambda_{i,\text{norm}}(u_i) u_i \\ &\text{where } \quad C = C_{\text{t}} = \infty. \end{aligned} \tag{12}$$

For convenience, we summarize the notation in Table I.

B. Performance analysis

Given these definitions, the following propositions hold.

Proposition 2: During the Normal network condition, over all pricing and CAC policy pairs, $(u_a^*, \pi_{u_a^*}^*)$ achieves the maximal commercial revenue performance subject to the constraint of meeting the loss rate requirements of public safety calls. Moreover, we have:

$$\begin{aligned} &R_{\text{norm}}(u, \pi_u^*)|_{C=C_{\text{t}}} \le R_{\text{norm}}(u_a^*, \pi_{u_a^*}^*)|_{C=C_{\text{t}}} \\ &\le R_{\text{norm}}(u_\infty)|_{C=\infty}. \end{aligned} \tag{13}$$

Proof
This follows from the definitions of $(u_a^*, \pi_{u_a^*}^*)$ and u_∞.

Proposition 3: We have the following relationship:

$$R_{\text{norm}}(u_c^*, \pi^g)|_{C=C_{\text{t}}} \le R_{\text{norm}}(u_b^*, \pi^g)|_{C=C_{\text{t}}}. \tag{14}$$

Proof
According to (9), given the greedy CAC policy, u_b^* is the optimal pricing for the loss rate unconstrained problem. Suppose $R_{\text{norm}}(u_c^*, \pi^g) > R(u_b^*, \pi^g)$, then $u_b^* \neq \arg\max_u R_{\text{norm}}(u, \pi^g)$ since u_c^* is a better solution for such a loss rate unconstrained problem. This is contradictory to our definition of u_b^*. Hence the result. *Q.E.D.*

We have shown so far that, combined with CAC policy $\pi_{u_a^*}^*$, u_a^* is the price vector that leads to the maximal revenue while satisfying the blocking (loss) rate requirements.

Following Proposition 2, we have the following relationship:

$$R_{\text{norm}}(u_b^*, \pi_{u_b^*}^*)|_{C=C_{\text{t}}} \le R_{\text{norm}}(u_a^*, \pi_{u_a^*}^*)|_{C=C_{\text{t}}}. \tag{15}$$

However, the computation of u_a^* is challenging, as we discuss in Section V-C. We therefore study the performance of using alternative prices numerically in Section VI. These numerical examples illustrate that in some practical settings, the performance loss of $R_{\text{norm}}(u_b^*, \pi^*_{u_b^*})$ compared with $R_{\text{norm}}(u_a^*, \pi^*_{u_a^*})$ is small. Meanwhile, the spectrum sharing through the combination of the pricing policy u_b^* and the CAC policy $\pi^*_{u_b^*}$ simplifies computation significantly. Additionally, the performance benefit of $R_{\text{norm}}(u_b^*, \pi^*_{u_b^*})$ over other pricing and CAC alternatives is large. Therefore, u_b^* is a sound price vector to use in practice.

Moreover, the numerical results in Section VI show that, in the absence of an optimal CAC in the second stage, the optimal prices have to be set high to reduce the traffic demand and thereby meet the loss rate requirements. However, with the use of an optimal CAC in the second stage, we can achieve a higher revenue and meet the call-level loss requirements simultaneously.

V. Computation Methods

A. Computation of prices u_∞^*, u_b^*, u_c^* and u_d^*

Given a demand function $\lambda(u)$, the computation of price u_∞^* is straightforward. In particular, for the linear arrival function $\lambda_i = k_{i,1} - k_{i,2} \cdot u_i$, used in Section VI, $u^*_{i,\infty} = \frac{k_{i,1}}{k_{i,2}}$.

Under the greedy CAC policy, its loss rate $L^g_{i,m}$ can be computed in close form using the method in [17]. Then, u_b^*, u_c^* and u_d^* can be computed according to (9), (10) and (11) , respectively, via standard optimization techniques.

B. Computation of optimal dynamic CAC

Per our system state definition in Section III-D, let $\boldsymbol{s}(t) := [n_1(t), ..., n_I(t), \omega(t)]$ be the system state at time t. The arrival rate distribution is a Poisson function with a mean arrival rate $\lambda_{i,m}$, and the holding times are exponentially distributed with a mean service rate μ_i for class-i traffic. For any admission control policy, the embedded Markov chain is recurrent and the policies are unichain. Consequently, $s(t)$ is an aperiodic and irreducible stationary Markov process over a finite state space. The state transitions and control selection take place at discrete times, but the time from one transition to the next is random. Hence a dynamic CAC policy can be modeled as the solution to a semi-Markov decision process (SMDP) problem [13]–[15], [18], [19]. The goal of the optimal dynamic CAC policy of our sharing mechanism is twofold: (a) maximize the average revenue performance over an infinite time horizon; and (b) satisfy the blocking (loss) rate requirements of the public safety calls. Thus, such a policy can naturally be modeled as the solution to a constrained continuous-time average reward SMDP problem.

We now describe the method for solving such an optimal dynamic CAC policy. The corresponding traffic parameters shall be used when solving the CAC policy for a particular network condition m. However, we drop the subscript m here for simple notation.

The detailed expressions of the key elements of this SMDP problem, namely, action space $\mathcal{A}(s)$, transition probabilities $p(s, a, s')$, state durations $\tau(s, a)$ and rewards $r(s, a)$ are included in Appendix I. It is shown in [12], [13], [20] that the CAC policy solution to such a constrained average-time SMDP problem is a randomized policy. In particular, let a_a denote the action of accept and a_r denote the action of reject, then the CAC policy accepts an arriving class-i call with a probability $\theta(s, a = a_a)$ and rejects the call with a probability $\theta(s, a = a_r) = 1 - \theta(s, a = a_a)$, for $s \in \mathcal{S}_i^A$ and $i \in \mathcal{I}_{\text{t}}$. Such a policy can be solved through a constrained linear programming (LP) formulation and techniques [12], [13], [20], [21], and the resulting solution is the optimal randomized CAC policy. Let $z_{s,a}$ denote the decision variables to the following constrained LP optimization problem in (16), where $z_{s,a}\,\tau(s, a)$ corresponds to the fraction of time spent in state s when action a is taken:

$$\max \sum_{s \in S} \sum_{a \in \mathcal{A}(s)} r(s, a)\; z_{s,a} \tag{16}$$

subject to

$$\begin{aligned} \sum_{a \in \mathcal{A}(s')} z_{s',a} &= \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}(s)} p(s, a, s')\; z_{s,a}, \\ \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}(s)} \tau(s, a)\; z_{s,a} &= 1, \\ z_{s,a} &\geq 0, \text{ for } s \in \mathcal{S},\ a \in \mathcal{A}(s), \\ \sum_{s \in \mathcal{S}_j^A} z_{s,a_r} - L_{j,\text{req}} \cdot \lambda_j &\leq 0,\ j \in \mathcal{I}_{\text{ps}}, \\ \sum_{s \in \mathcal{S}_j^A} z_{s,a_r} - L_{j,\text{req}} \cdot \lambda_j \leq 0,\ L_{j,\text{req}} &= 1,\ j \in \mathcal{I}_{\text{com}}. \end{aligned}$$

Per Proposition 1, this LP problem is feasible. Subsequently, the probability $\theta(s, a_a)$ of taking action a_a at state s is expressed as: $\theta(s, a_a) = \frac{z_{s,a_a}}{z_{s,a_a} + z_{s,a_r}}$. At any state s, the desired optimal randomized CAC policy is to take action a_a with probability $\theta(s, a_a)$, take action a_r with probability $1 - \theta(s, a_a)$ if $z_{s,a_a} + z_{s,a_r} > 0$, and to take an action arbitrarily if $z_{s,a_a} + z_{s,a_r} = 0$.

C. Computation of price u_a^*

It is challenging to find the joint optimal pricing and CAC u_a^* and $\pi^*_{u_a^*}$ analytically. Although we have described the computational method to derive π_u^* for a given price vector u in Section V-B, the solution cannot be expressed in closed form. As a result, u_a^* cannot be computed as prices (e.g. u_b^*, u_c^*, etc.). For the case where the problem is small, e.g., $|\mathcal{I}_{\text{com}}| = 1$, we can solve the problem through a brute-force computation and approximation. Specifically, we can generate a price grid that covers the entire price range $[0, u_{i,\max}]$ with a fine granularity, and identify the corresponding constrained CAC policy π_u^* for each price point during the Normal network condition. The price and CAC pair leading to the maximal revenue is the approximation of the $(u_a^*, \pi^*_{u_a^*})$ pair.

VI. Numerical Examples

In this section, we provide numerical illustrations of the performance and properties of our spectrum sharing mechanism.

TABLE II
RELATIVE TRAFFIC ARRIVAL RATES UNDER DIFFERENT NETWORK CONDITIONS

Network conditions	Public safety calls $i \in \mathcal{I}_{\text{ps}}$	Commercial calls $i \in \mathcal{I}_{\text{com}}$
Normal	$\lambda_{i,\text{norm}}$	$\lambda_{i,\text{norm}}(u_i)$
Emergency	$10 \cdot \lambda_{i,\text{norm}}$	$\lambda_{i,\text{norm}}(u_i)$
Disaster	$10 \cdot \lambda_{i,\text{norm}}$	$10 \cdot \lambda_{i,\text{norm}}(u_i)$
Public Event	$\lambda_{i,\text{norm}}$	$10 \cdot \lambda_{i,\text{norm}}(u_i)$

A. Network conditions

In principle, traffic load for public safety users and commercial users can vary independently between normal and high usage. We examine four extreme network conditions where $\mathcal{M} = \Big\{m \mid m \in \{\text{norm}, \text{emer}, \text{dstr}, \text{evnt}\}\Big\}$ is the set of prototypical conditions. The characteristics of these network conditions are defined in Section III-B.

We assume that public safety call classes have a predefined fixed arrival rate $\lambda_{i,\text{norm}}$ during the Normal condition for $i \in \mathcal{I}_{\text{ps}}$. Additionally, we capture the traffic loads during special conditions, such as the usage surge during Emergency and Disaster, by:

$$\begin{aligned}\lambda_{i,\,\text{evnt}} &= \lambda_{i,\,\text{norm}},\\ \lambda_{i,\,\text{emer}} &= 10 \cdot \lambda_{i,\,\text{norm}},\\ \lambda_{i,\,\text{dstr}} &= 10 \cdot \lambda_{i,\,\text{norm}},\\ \text{where} \quad & i \in \mathcal{I}_{\text{ps}}.\end{aligned}$$

For commercial users, we assume the following demand functions for Normal and Emergency conditions:

$$\begin{aligned}\lambda_{i,m} &= \lambda_i(u_i), \qquad (17)\\ \text{where} \quad u_i &\in [\,0, u_{i,\max}\,],\\ i &\in \mathcal{I}_{\text{com}},\\ m &\in \{\text{norm}, \text{emer}\}.\end{aligned}$$

Similarly, we model the surge of service demand during Disaster and Public Event by:

$$\begin{aligned}\lambda_{i,m} &= 10 \cdot \lambda_i(u_i), \qquad (18)\\ \text{where} \quad u_i &\in [\,0, u_{i,\max}\,],\\ i &\in \mathcal{I}_{\text{com}},\\ m &\in \{\text{dstr}, \text{evnt}\}.\end{aligned}$$

Therefore, the arrival rates of both public safety and commercial calls $\lambda_{i,m}$ are a function of the network condition. Their relative traffic arrival rates are summarized in Table II.

B. Example parameters

For segregated networks, let the capacities dedicated to public safety calls and commercial usage be $C_{\text{ps}} = 2$ and $C_{\text{com}} = 9$, respectively; for the shared network, it is $C_t = C_{\text{ps}} + C_{\text{com}} = 11$. Additionally, we assume that there is one class of public safety calls and one class of commercial calls in the shared network. For simplicity, we call them class 1 and class 2, respectively. We also assume that both classes require one unit of transmission bandwidth, i.e., $x_1 = x_2 = 1$. The service rates are $\mu_1 = 3$ and $\mu_2 = 1$ per unit time, and the blocking (loss) rate constraint for public safety calls is less than or equal to 1%, i.e., $L_{1,m} \leq 1\%$, $m \in \mathcal{M}$.

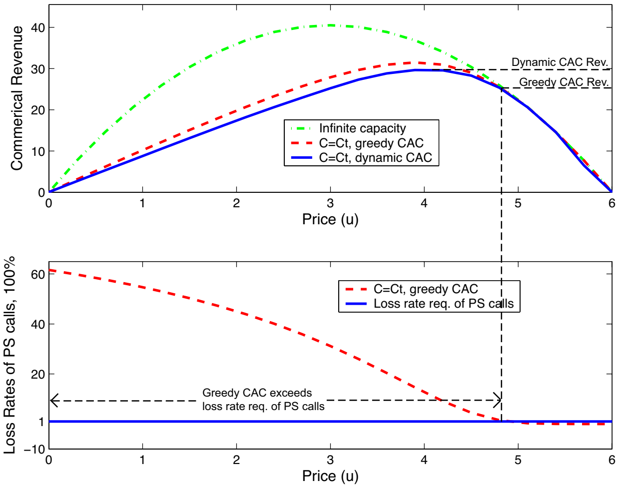

Fig. 1. Commercial revenue and public safety call blocking performance in the Normal condition, Demand Function I.

The arrival rate of public safety calls during the Normal condition is $\lambda_{1,\text{norm}} = 0.45$ per unit time. The arrival rates of the commercial calls (i.e. calls from class 2) follow a linear demand function. Specifically, we study the performance when the demand functions are in two different forms, denoted as DF-I and DF-II and expressed by (19) and (20), respectively. The first tends to overload the network, whereas the second tends to underload it.

$$\text{DF-I:} \quad \lambda_{2,\text{norm}} = 27 - 4.5 \cdot u_2, \qquad u_2 \in [0, 6], \quad (19)$$

$$\text{DF-II:} \quad \lambda_{2,\text{norm}} = 7.75 - 0.65 \cdot u_2, \quad u_2 \in [0, 12.24]. \quad (20)$$

C. Numerical results

For both demand functions, we first investigate the commercial revenue and public safety loss rate performance as a function of underlying prices and CAC policies in the Normal condition, and present their results in Figures 1 and 2, respectively. In the upper plots, the dash-dot curves represent the computed revenue performance when the capacity of the shared network is infinite, the dash curves are the computed revenue performance with the greedy CAC policy when the capacity of the network is $C = C_{\text{t}}$, and the solid curves are the simulated revenue performance with the optimal constrained dynamic CAC, generated using the method in Section V-B. In the lower plots, the dash curves are the computed blocking (loss) rates of the public safety calls assuming the greedy CAC, plotted against the loss rate requirement of public safety calls, $L_{1,\text{req}} = 1\%$, depicted in the solid lines.

Next, for both demand functions, we study the performance of the shared network in all network conditions at the pricing points defined in Section IV-A. We identify the various optimal pricing and present the results in Table III. In particular, in the brute-force computation for u_a^*, the price resolution is 5% of the corresponding $u_{2,\max}$ (i.e., 0.3 for DF-I and 0.60 for DF-II). We then apply these prices and generate performance results. Specifically, for the segregated public safety or commercial networks, since the optimal CAC policy is the greedy

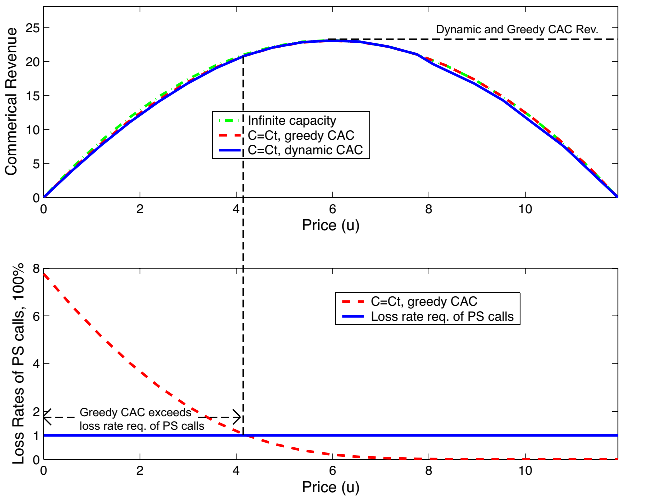

Fig. 2. Commercial revenue and public safety call blocking performance in the Normal condition, Demand Function II.

policy, we compute their corresponding loss rates in a closed form, and subsequently compute the revenue performance, as described in Section V-A. For the shared networks, at each price point, we generate the greedy CAC policy and solve the optimal constrained dynamic policy according to the method in Section V-C, and then simulate the performance of these CAC policies with extended network simulations.[3]

For the Public Event condition with DF-I, we repeat ten test runs, where each run is a simulation over 10 million traffic events (i.e., call arrivals or departures). For all other cases, we repeat five test runs, where each run is a simulation of over 3 million traffic events.[4] We present the performance mean values in Tables IV–VII. The standard deviations of loss rates are less than 8% of the corresponding mean values for all reported loss rate results, and the standard deviations for the revenue performance values are less than 0.13% of their reported mean values.

D. Discussion

Our numerical results illustrate that regardless of the applied CAC policy, the underlying prices affect the commercial revenue performance significantly, as shown in Figures 1 and 2. This demonstrates that the selection of service prices is important for the network revenue. With the use of a sound price (e.g., u_b^*), our sharing strategy obtains significant revenue performance gain compared to a scheme with an arbitrarily chosen non-optimal price.

At the optimal pricing point given the greedy CAC policy (i.e., when $u = u_a^*$), the loss rate of public safety calls may not meet the loss rate requirement. In the case of DF-I, such a requirement is not met in any network condition; in the case of DF-II, it is only met during the Normal condition, as shown in Figures 1 and 2, and Tables IV and VI. However, with the optimal constrained dynamic CAC policy, similar

[3]The performance of the greedy CAC in the shared network can also be computed, as is done for the segregated networks.

[4]Due to the low arrival rates of public safety calls, longer simulations were run for the cases in Public Event condition with DF-I to achieve a sound statistics average.

TABLE III
COMMERCIAL PRICE RESULTS

Demand function	u^{∞}	u_a^*	u_b^*	u_c^*	u_d^*
DF-I	3.00	3.90	3.91	4.89	4.12
DF-II	5.96	5.96	6.00	6.00	6.12

revenue performance is achieved while meeting the loss rate requirement of public safety calls (within a standard deviation of the measurement error).

In addition, with the optimal constrained dynamic CAC policy, the loss rate of public safety calls meets the requirement in all cases, regardless of the underlying price, as shown in Tables IV and VI. By contrast, such a loss rate requirement is not met in the segregated public safety network when the network is in the Disaster and the Public Event conditions, as shown in Tables IV and VI. To meet the loss rate requirement in all network conditions, the capacity of the segregated public safety network needs to expand from $C_{\text{ps}} = 2$ to $C_{\text{ps}} = 6$.

The optimal prices u_a^* in a shared network are less than the optimal prices in the segregated commercial network u_d^* for both demand functions, as shown in Table III. This is beneficial to the commercial network users since their cost of wireless communication is reduced in a shared network. Moreover, Table III also shows that u_b^* is close to u_a^* for both demand functions, which indicates that the revenue performance resulting from the pricing and CAC pair $(u_b^*, \pi^*_{u_b^*})$ is close to the revenue obtained via the pricing and CAC pair $(u_a^*, \pi^*_{u_a^*})$. This suggests that u_b^* can be used as the optimal pricing in practice since it is easier to compute than u_a^*. The intuition behind this phenomenon is that, in the Normal condition, since the overall load of the public safety calls is far less than the overall load of the commercial calls, $\pi^*_{u_a^*}$ is close to π^g in spite of the fact that $\pi^*_{u_a^*}$ needs to ensure the meeting of the loss rate requirement of public safety calls. Consequently, u_a^* and u_b^* are close.

The revenue performance via the combination of a good price (e.g., u_b^*) and the optimal constrained dynamic CAC is higher than the revenue achieved via the optimal pricing in the segregated commercial network, as shown in Table V and Table VII. This result shows that our sharing strategy is beneficial to the commercial network service provider.

Finally, we observe that different users' behaviors as a function of the price, as reflected by different demand functions, play an important role in the network performance. The three revenue-prices curves are distinguished for the case of DF-I as depicted in Figure 1, but almost overlap for the case of DF-II as shown in Figure 2. The overall traffic load is heavier in the case of DF-I than that of DF-II, as shown in Figures 1 and 2, and Tables IV and VI.

VII. IMPLICATIONS OF RESULTS AND EXTENSIONS

Our results have shown that the benefits of our spectrum sharing strategy for the public safety and commercial communications are threefold:

1) Relative to the dedicated spectrum assignment for public safety calls to provide a guaranteed low blocking probability, our sharing strategy uses spectrum resource more efficiently by meeting the objectives of public

TABLE IV
LOSS RATE PERFORMANCE, DEMAND FUNCTION I

	CAC	Segregated network		Shared network					
		$u_d^* = 4.12$		$u_\infty^* = 3.00$		$u_a^* \cong u_b^* = 3.91$		$u_c^* = 4.89$	
		PS 100%	Com 100%	PS 100%	Com 100%	PS 100%	Com 100%	PS 100%	Com 100%
Normal	Greedy	0.97	19.79	31.23	31.09	14.36	14.30	0.98	1.00
Emergency		31.03	19.79	35.87	35.85	20.14	20.08	3.46	3.46
Disaster		31.03	89.52	91.97	92.01	88.60	88.61	79.21	79.19
Public Eve.		0.97	89.52	91.92	91.93	88.49	88.44	78.59	78.63
Normal	Dynamic	—	—	0.99	37.69	1.02	19.33	1.00	1.00
Emergency		—	—	0.99	54.70	0.99	38.36	1.03	8.49
Disaster		—	—	1.02	95.17	1.00	93.04	1.01	87.07
Public Eve.		—	—	1.02	93.24	0.99	90.30	1.00	81.96

TABLE V
REVENUE PERFORMANCE, DEMAND FUNCTION I

CAC Policy	Segregated network	Shared network		
	$u_d^* = 4.12$	$u_\infty^* = 3.00$	$u_a^* = u_b^* = 3.91$	$u_c^* = 4.89$
Greedy	28.00	27.91	31.49	24.24
Dynamic	—	25.23	29.64	24.24

TABLE VI
LOSS RATE PERFORMANCE, DEMAND FUNCTION II

	CAC	Segregated network		Shared network			
		$u_d^* = 6.12$		$u_\infty^* = u_a^* = 5.96$		$u_b^* \cong u_c^* = 6.00$	
		PS 100%	Com 100%	PS 100%	Com 100%	PS 100%	Com 100%
Normal	Greedy	0.97	0.98	0.20	0.20	0.20	0.20
Emergency		31.03	0.98	1.27	1.28	1.24	1.25
Disaster		31.03	76.90	72.81	72.86	73.40	73.36
Public Eve.		0.97	76.90	71.98	71.90	72.42	72.46
Normal	Dynamic	—	—	1.02	0.20	1.03	0.20
Emergency		—	—	1.01	1.85	1.02	1.73
Disaster		—	—	0.99	82.89	1.00	83.24
Public Eve.		—	—	0.97	76.16	0.99	76.66

TABLE VII
REVENUE PERFORMANCE, DEMAND FUNCTION II

CAC Policy	Segregated network	Shared network	
	$u_d^* = 6.12$	$u_\infty^* = u_a^* = 5.96$	$u_b^* = u_c^* = 6.00$
Greedy	22.86	23.07	23.06
Dynamic	—	23.07	23.06

safety and commercial communications simultaneously. The public safety calls can meet their requirements in more conditions without requiring additional dedicated spectrum.

2) Our combined pricing and CAC control not only enables the meeting of the loss rate requirements of public safety calls in all network conditions, but also achieves good revenue performance compared to the revenue obtained via arbitrarily chosen non-optimal prices. Moreover, relative to the segregated network, our approach leads to a lower price to commercial users, and a higher revenue for the network provider even compared to using the optimal price in the segregated commercial network.
3) Our approach is feasible. Our notion of optimal pricing is designed according to the traffic statistics of the Normal network condition, and this single optimal pricing is applied in all network conditions (e.g., Emergency, Disaster, and Public Event). Different optimal constrained dynamic CAC is applied to each network traffic condition. Because changing prices must be communicated to the network customers, and the change of CAC is transparent to the customers, our approach can react quickly to changing network condition as well as ensure the stability of the network operation.

Future work will include the study of coordinating spectrum sharing between different technologies or with distributed control, the investigation of CAC policies that allow preemptive dropping of commercial calls, continuous policy adaptation, and providing bounds on the performance differences of various combinations of pricing and CAC policies.

VIII. CONCLUSION

We have investigated the problem of spectrum sharing between public safety and commercial communications. We have provided a precise characterization of the performance objectives of such a sharing strategy, which is to satisfy the public safety communication requirement of a high availability (i.e., low blocking) in all network conditions and the commercial communication requirement of a high network revenue. Subsequently, we have proposed a spectrum sharing strategy via network pricing and call admission control. We have demonstrated through both analysis and simulation that, relative to the segregated networks using the same amount of spectrum resource, our sharing strategy not only satisfies the objective of public safety communication, but also achieves superior commercial revenue performance, at lower prices to consumers.

APPENDIX I SMDP ELEMENTS

The key elements of our SMDP (i.e., CAC) problem: *state, action space, transition probability, state duration* and *reward*, are described as follows.

- *State*

  The combination of a configuration and a traffic event constitute a state given by (1) in Section III-D.
- *Action space*

  When a new class i call arrives and requests to enter the system, the network controller can either accept or reject the connection request. If the call is accepted, the system transitions from the current configuration n immediately to the next configuration $n' = n + \omega$. Otherwise, the system stays at the same configuration, that is $n' = n$. When a call served in the system requests to depart, the network controller always accepts such a request and the system subsequently transitions to the next configuration $n' = n + \omega$. Given the current state and action, the system deterministically transitions to configuration n' and subsequently transitions to the next state upon the next traffic event.

  Let $\mathcal{A}(s)$ denote the set of actions a at state s, and let $a = a_a$ and $a = a_r$ denote the acceptance and rejection of a call request, respectively. Then the action space can be summarized as follows.

$$\mathcal{A}(s) = \begin{cases} \{a_a, a_r\}\ , & \omega = \boldsymbol{e}_i, \quad i = 1, ..., I \\ \{a_a\} & \omega = -\boldsymbol{e}_i, \quad i = 1, ..., I \end{cases} \tag{21}$$

- *Transition probability*

  Due to our state and action definition for our particular SMDP (i.e., CAC) problem, upon an action taken at the time instant of the current traffic event, the system will stochastically transition from the current state s to the next state s'. In particular, the transition to state s' will take place at the time instant of the next traffic event. Let $p(s, a, s')$ denote the transition probabilities from state s to s' when an action a is taken. First, we define $\nu(n)$ as the transition rate at configuration n:

$$\nu(n) = \sum_{i=1}^{I} \lambda_i + \sum_{i=1}^{I} n_i \mu_i \tag{22}$$

  When $s = (n, \boldsymbol{e}_i)$ and $a = a_r$,

$$p(s, a, s') = \begin{cases} \frac{\lambda_j}{\nu(n)}\ , & s' = (n, \boldsymbol{e}_j) \\ \frac{n_j\ \mu_j}{\nu(n)}\ , & s' = (n, -\boldsymbol{e}_j) \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

  When $s = (n, \boldsymbol{e}_i)$ and $a = a_a$,

$$p(s, a, s') = \begin{cases} \frac{\lambda_j}{\nu(n+\boldsymbol{e}_i)}\ , & s' = (n + \boldsymbol{e}_i, \boldsymbol{e}_j) \\ \frac{(n_j + \delta(i-j))\ \mu_j}{\nu(n+\boldsymbol{e}_i)}\ , & s' = (n + \boldsymbol{e}_i, -\boldsymbol{e}_j) \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

  When $s = (n, -\boldsymbol{e}_i)$ and $a = a_a$,

$$p(s, a, s') = \begin{cases} \frac{\lambda_j}{\nu(n-\boldsymbol{e}_i)}\ , & s' = (n - \boldsymbol{e}_i, \boldsymbol{e}_j) \\ \frac{(n_j - \delta(i-j))\ \mu_j}{\nu(n-\boldsymbol{e}_i)}\ , & s' = (n - \boldsymbol{e}_i, -\boldsymbol{e}_j) \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

- *State duration*

  The duration between state transitions has the following probability distribution.

$$F_{ss'}(\xi|a) = \int_0^{\xi} \zeta_s(a) e^{-\zeta_s(a)t}\ t\ dt \tag{26}$$

  where

$$\zeta_s(a) = \begin{cases} \nu(n)\ , & s = (n + \boldsymbol{e}_i),\ a = a_r \\ \nu(n + \boldsymbol{e}_i)\ , & s = (n + \boldsymbol{e}_i),\ a = a_a \\ \nu(n - \boldsymbol{e}_i)\ , & s = (n - \boldsymbol{e}_i),\ a = a_a \end{cases} \tag{27}$$

  and $\zeta_s(a)$ is uniformly bounded in the sense that

$$\zeta_s(a) \le \max_{s,a} \zeta_s(a) \qquad s \in \mathcal{S},\ a \in \mathcal{A}$$

- *Reward*

  We define $r(s, a)$ as the expected reward of taking an action a at state s. Since we assume that whenever a class-i call is admitted to the network, the network immediately collects a revenue and there is no other cumulative reward to be gained during the call service time, and no revenue will be collected for rejected calls, $r(s, a)$ can be expressed by (28).

$$r(s, a) = \begin{cases} u_i\ , & \text{if } \omega = \boldsymbol{e}_i \text{ and } a = a_a \\ 0\ , & \text{otherwise.} \end{cases} \tag{28}$$

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